Coriolis force

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(Dated: July 17, 2020)

"I have no satisfaction in formulas unless I feel their arithmetical magnitude."
— Baron William Thomson Kelvin (1824-1907)

In earlier lecture notes, we have discussed about reference frames and their types. Also discussed transformation equations between two frames (one frame inertial and other rotating with an angular velocity ω). In Fig (1), we consider that S frame is inertial and S' is rotating with an angular velocity ω. We have also considered that both the frames have common origin O and axis of rotation for both the frames are passing through O. Next, we consider a moving particle P. The value of position vector $\mathbf{r}$ will be same for both the frames though its components will be different. Note that we have already seen that length is invariant in inertial and the rotating frame.

Next, we we consider that the particle be at rest w.r.t. S-frame, its relative linear velocity will be $-\boldsymbol{\omega}\times\mathbf{r}$ in the S'-frame which is the rotating frame and is non-inertial. Now if we take that particle has a linear velocity $\mathbf{v} = d\mathbf{r}/dt$ in the S-frame, for the S' frame the velocity $\mathbf{v}'$ will be given by

$$\mathbf{v}' = \frac{d\mathbf{r}}{dt} - \boldsymbol{\omega}\times\mathbf{r}\ . \tag{1}$$

Now we can write

$$\left[\frac{d\mathbf{r}}{dt}\right]_{S'} = \left[\frac{d\mathbf{r}}{dt}\right]_{S} - \boldsymbol{\omega}\times\mathbf{r} \tag{2}$$

which is equivalent to say

$$\mathbf{v}' = \mathbf{v} - \boldsymbol{\omega}\times\mathbf{r}\ . \tag{3}$$

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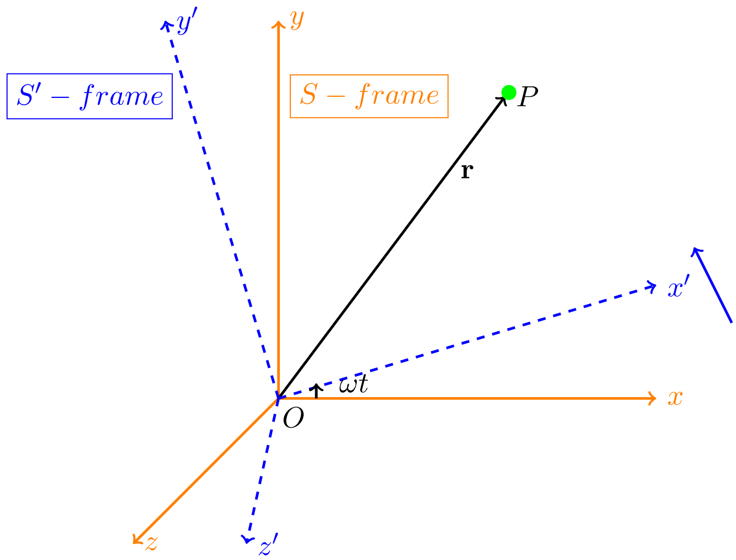

FIG. 1: Rotating frame.

For S-frame

$$\left[\frac{d\mathbf{r}}{dt}\right]_S = \left[\frac{d\mathbf{r}}{dt}\right]_{S'} + \boldsymbol{\omega}\times\mathbf{r} \tag{4}$$

or

$$\mathbf{v} = \mathbf{v}' + \boldsymbol{\omega}\times\mathbf{r}\ . \tag{5}$$

We can write Eq. (4) in operator form as

$$\left[\frac{d}{dt}\right]_S = \left[\frac{d}{dt}\right]_{S'} + \boldsymbol{\omega}\times \quad . \tag{6}$$

Next, for velocity vector $\mathbf{v}$, we can write, using Eq. (6)

$$\left[\frac{d\mathbf{v}}{dt}\right]_S = \left[\frac{d\mathbf{v}}{dt}\right]_{S'} + \boldsymbol{\omega}\times\mathbf{v} \tag{7}$$

or we can remove the subscripts S and S', which should be understood, and write above equation as

$$\frac{d\mathbf{v}}{dt} = \frac{d\mathbf{v}'}{dt} + \boldsymbol{\omega}\times\mathbf{v}\ . \tag{8}$$

Next, substituting value of $\mathbf{v}$ in Eq. (7)

$$\left[\frac{d\mathbf{v}}{dt}\right]_S = \frac{d}{dt}\Big[(\mathbf{v}' + \boldsymbol{\omega}\times\mathbf{r})\Big]_{S'} + \boldsymbol{\omega}\times(\mathbf{v}' + \boldsymbol{\omega}\times\mathbf{r}) \tag{9}$$

or

$$\frac{d\mathbf{v}}{dt} = \frac{d\mathbf{v}'}{dt} + \frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} + \boldsymbol{\omega}\times\left[\frac{d\mathbf{r}}{dt}\right]_{S'} + \boldsymbol{\omega}\times\mathbf{v}' + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})\ . \tag{10}$$

Here, it is to be noted that $d\mathbf{v}/dt$ is acceleration $\mathbf{a}$ (say) observed by S frame observer and $d\mathbf{v}'/dt$ is the acceleration $\mathbf{a}'$ (say)observed by S'-frame observer. Since angular velocity is uniform so we take $d\boldsymbol{\omega}/dt = 0$. Now we can write

$$\mathbf{a} = \mathbf{a}' + 2\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad , \tag{11}$$

where we have used the definition $[d\mathbf{r}/dt]_{S'} = \mathbf{v}'$. In Eq. (11) $2\boldsymbol{\omega} \times \mathbf{v}'$ is *Coriolis acceleration* after **G. Coriolis** who first identified this and $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ is the *centripetal acceleration.* Therefore, we can write

observed acceleration in inertial frame =observed acceleration in rotating frame +Coriolis acceleration +centripetal acceleration.

In terms of force Eq. (11) is written as

$$m\mathbf{a} = m\mathbf{a}' + 2m\boldsymbol{\omega} \times \mathbf{v}' + m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \tag{12}$$

or

$$\mathbf{F} = \mathbf{F}' + 2m\boldsymbol{\omega} \times \mathbf{v}' + m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad . \tag{13}$$

Therefore, force observed in S'-frame is given by

$$\mathbf{F}' = \mathbf{F} - 2m\boldsymbol{\omega} \times \mathbf{v}' - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad . \tag{14}$$

But we know that force acting on the particle in a non-inertial frame is given by

$$\mathbf{F}' = \mathbf{F} + \mathbf{F}_0 \ , \tag{15}$$

where $\mathbf{F}_0$ is the *fictitious force.* Therefore, the *fictitious force* $\mathbf{F}_0$ is identified as

$$\mathbf{F}_0 = -2m\boldsymbol{\omega} \times \mathbf{v}' - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad , \tag{16}$$

where $-2m\boldsymbol{\omega} \times \mathbf{v}'$ is identified as Coriolis force and $-m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ is the centripetal force. Therefore, we can write

fictitious force =Coriolis force + centripetal force.

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[1] D.S. Mathur, *Mechanics*, S. Chand & Company Ltd., New Delhi, (2007) .